Probabilistic Timing Estimates in Scenarios Under Testing Constraints

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Agenda

1. Probabilistic Timing Analysis (PTA): Motivation and Problem Statement
2. Measurement-Based PTA under Testing Constraints
3. Uncertainty in Extreme Value Theory (EVT)
4. Markov's Inequality for pWCET in Low Sample Scenarios
5. Results
6. Conclusions

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Motivation

Soft Real-Time Requirements in Modern MPSoCs

- Multi-Processor Systems on Chips (MPSoCs) are becoming more complex
- Timing Analysis (TA) is difficult due to complex software and hardware
- Measurement-Based Probabilistic Timing Analysis (MBPTA) provides solid TA

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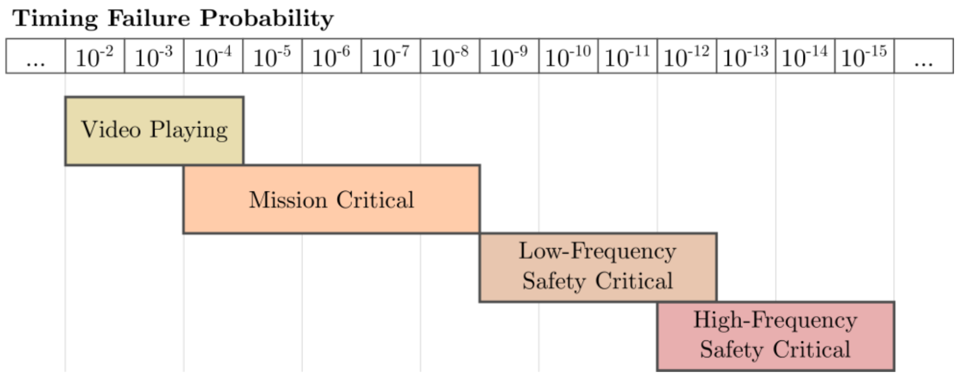
- Timing Analysis (TA) is difficult due to complex software and hardware
- Measurement-Based Probabilistic Timing Analysis (MBPTA) provides solid TA
- Risk tolerance shall be assessed against practicality
- More precise estimations for timing are in proportion to the time and effort required to apply MBPTA techniques
- In MBPTA this is directly related to the number of test/runs (sample size)
  - **Costs:** increasing number of tests may be too costly (e.g. on-the-road testing)
  - **Benefits:** timing can range from quality, to economic loss or causalities
- However, low sample sizes can produce high uncertainty



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Probabilistic Timing Analysis /1

Measurement-Based Probabilistic
Timing Analysis

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Probabilistic Timing Analysis /1

Measurement-Based Probabilistic Timing Analysis

- Based directly on the test/runs on the hardware and application
- The goal is to derive a probabilistic upperbound on the probabilistic Worst-Case Execution Time (pWCET)
- We aim at low sample sizes (N = 100)
- The CCDF denotes the probability of exceeding a certain value (e.g. ET)
- The pWCET required properties
  - ✗ **Optimistic**:
    - ✗ $p_{bound} < p_{real}$
    - ✗ $et_{bound} < et_{real}$
  - ✓ **Conservative**:
    - ✓ $p_{bound} \geq p_{real}$
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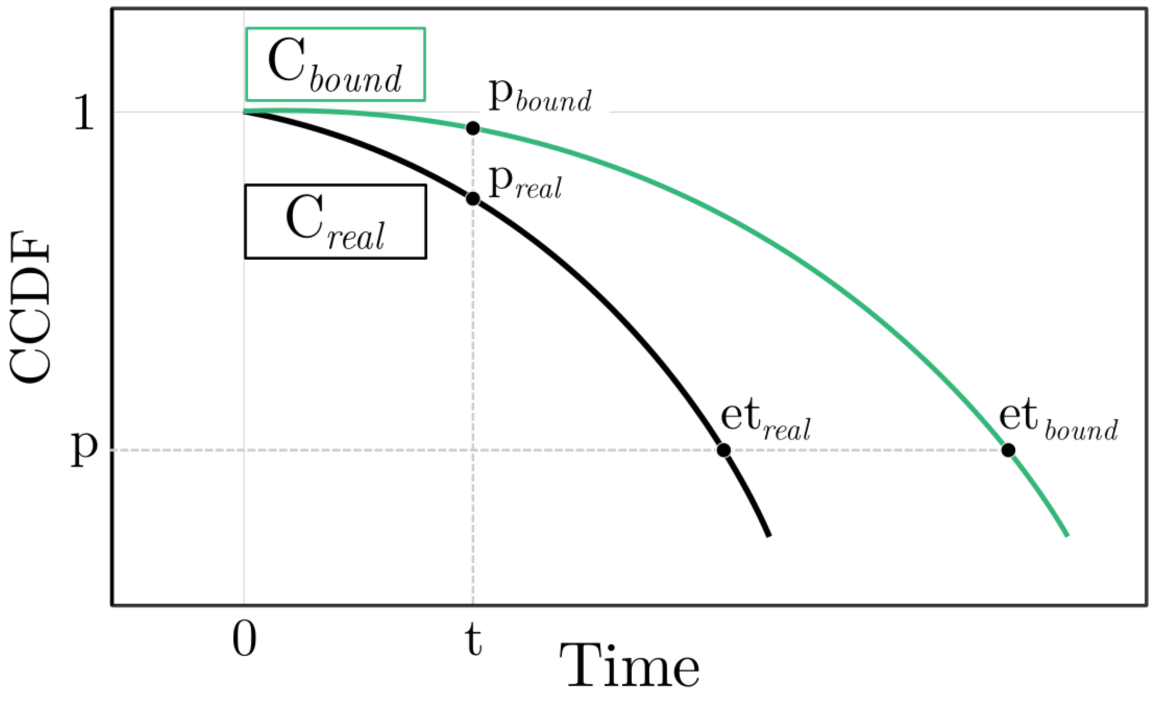

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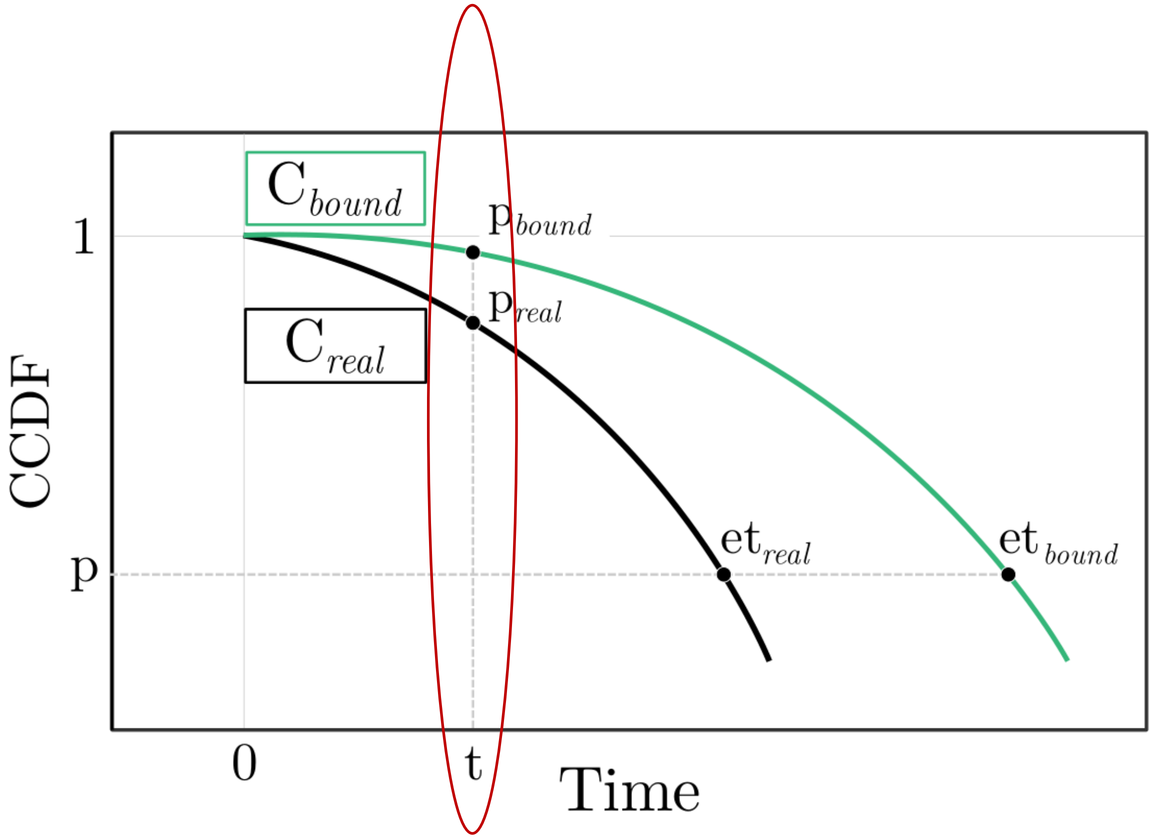

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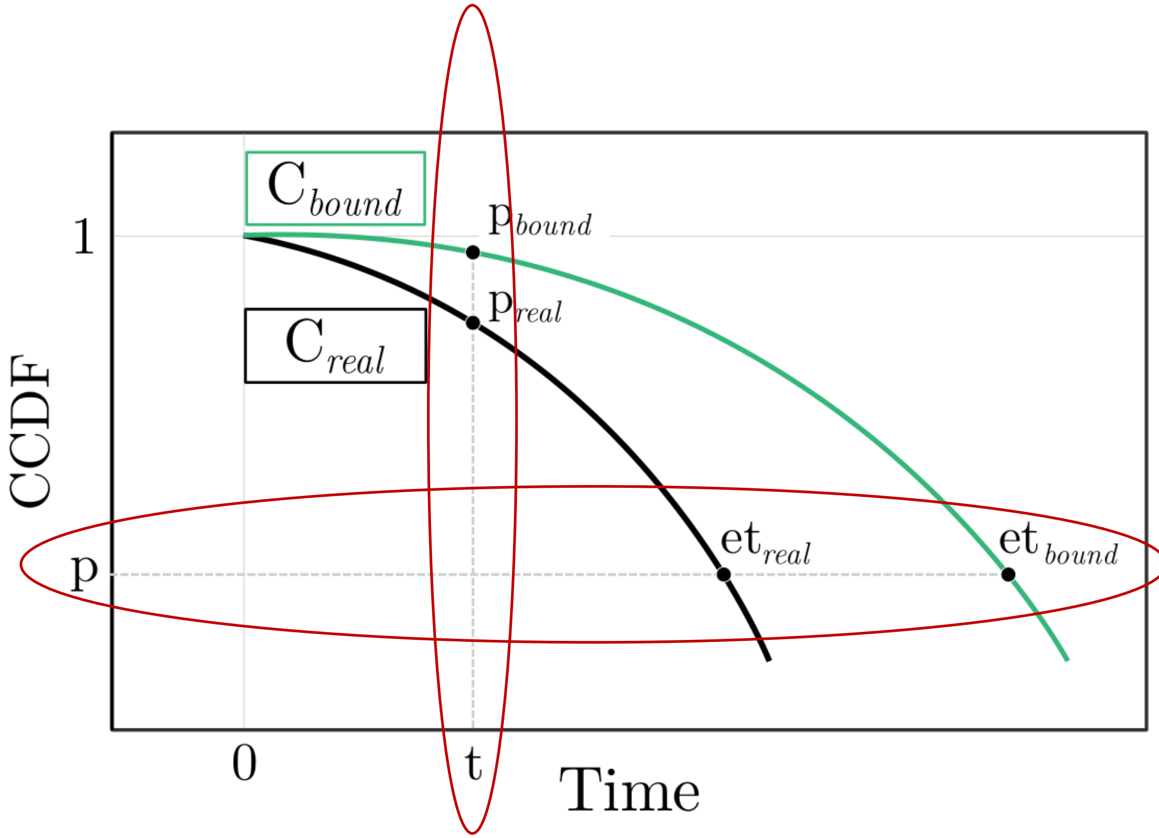

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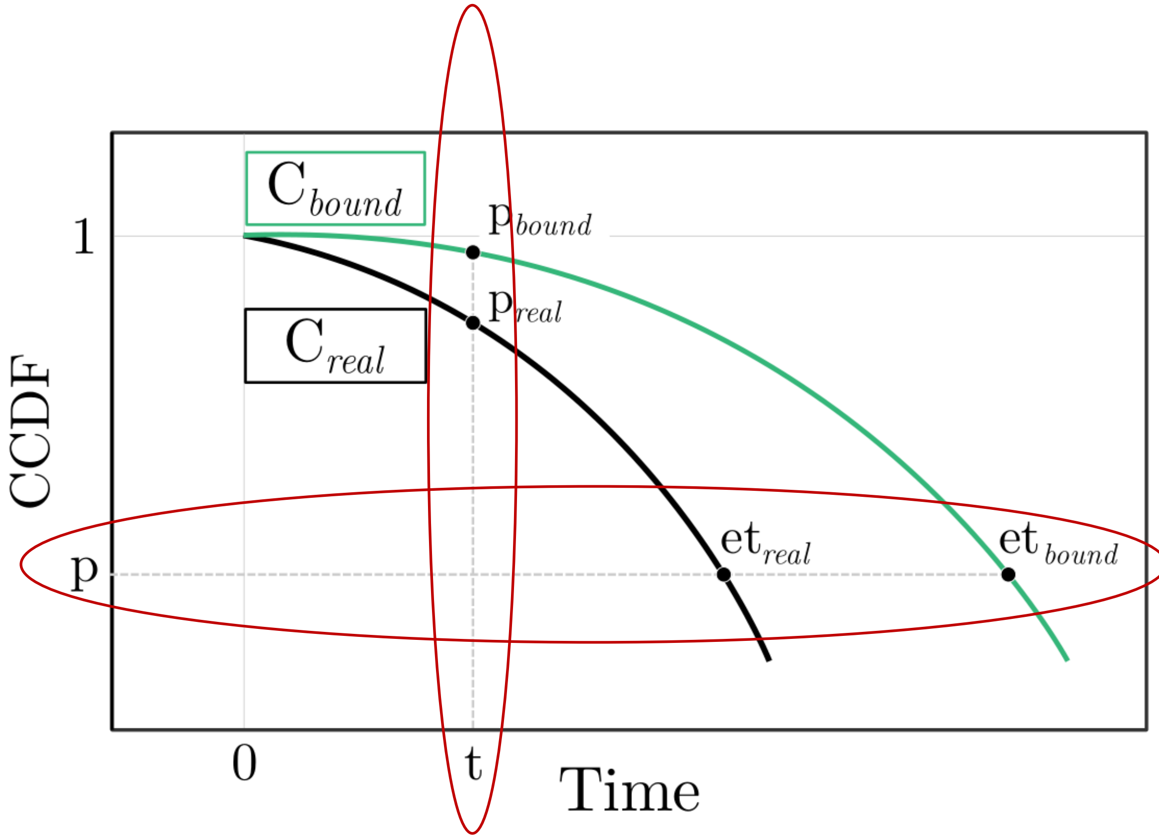
- Exceedingly pessimistic pWCET are not useful
- pWCET estimates should be **tight** to the real distribution



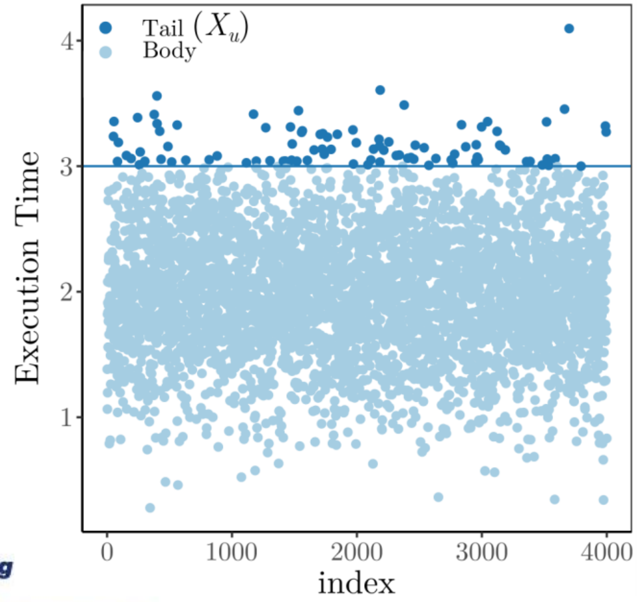
$$tightness(p) = \frac{et_{bound}(p)}{et_{real}(p)}$$

Probabilistic Timing Analysis /2

- Two main frameworks to derive pWCET are used in this work
  - **Extreme Value Theory:** based on the asymptotic distribution of the highest quantiles. Estimating the **extreme value index ξ is crucial**.

EVT

$$G(y;\sigma,\xi) = \begin{cases} 1-\left(1+\xi\frac{y}{\sigma}\right)^{-\frac{1}{\xi}} & \text{if} \quad \xi \neq 0, \\ 1-\exp\left(-\frac{y}{\sigma}\right) & \text{if} \quad \xi = 0, \end{cases}$$



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SAFEXPLAIN

Probabilistic Timing Analysis /2

- Two main frameworks to derive pWCET are used in this work
  - **Extreme Value Theory:** based on the asymptotic distribution of the highest quantiles. Estimating the **extreme value index ξ is crucial**.
  - **Markov's Inequality:** based on a probabilistic upperbound of the moments of a distribution

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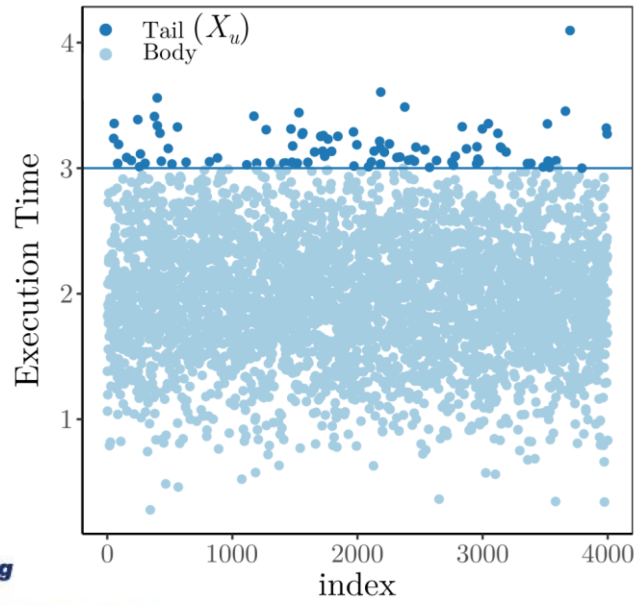
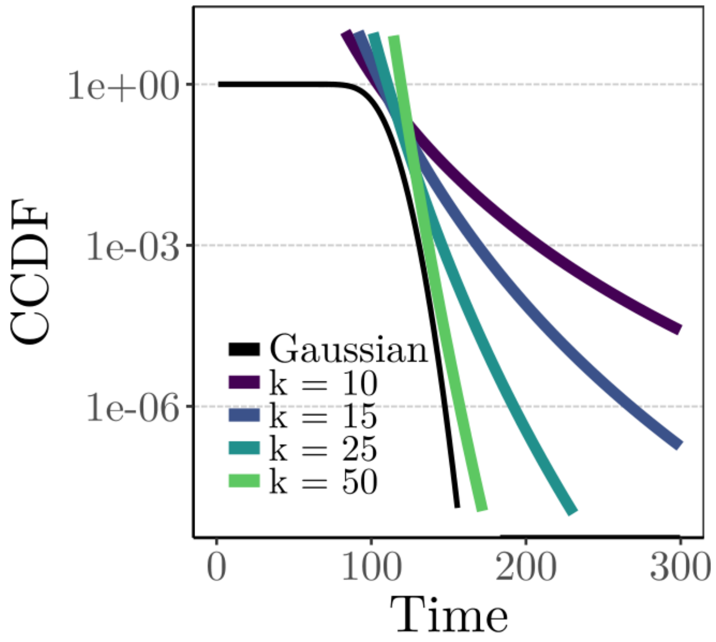


Markov

$$P(X \geq b) \leq \frac{E(X^k)}{b^k}$$



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EVT and Uncertainty Estimation

EVT Models

- We use three methods to **estimate the tail**, for EXP and GPD models, with different approaches for modelling diversity

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CV Plot

$$\widehat{\text{cv}}_u = \frac{\sqrt{\hat{V}(X - u|x > u)}}{\hat{E}(X - u|x > u)},$$

Residual Coefficient of Variation

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Quantile-Quantile

$$d_b(u) = \frac{1}{m}\sum_{j=1}^{m}\left|\frac{\hat{\sigma}_u^b}{\hat{\xi}_u^b}\left[(1-p_j)^{\hat{\xi}_u^b}-1\right]-Q(p_j,X_u^b)\right|,$$

Quantile distance

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Quantile distance

Semi-Parametric

$$L_{glt}^{PoT}(X) = L_{bulk}(X)(1-H(u|\gamma))L_{glt}(X)$$

$$= \underbrace{\prod_{x_i \le u} h(x_i|\gamma)}_{\text{Semi-parametric}} \underbrace{\prod_{x_i > u} \frac{1-H(u|\gamma)}{\sigma}\left(1+\xi\frac{x_i-u}{\sigma}\right)^{-\frac{(1+\xi)}{\xi}}}_{\text{Parametric}},$$

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Uncertainty Analysis of EVT Models

- For the **Semi-parametric and the QQ models**, we need to assess the uncertainty of the estimator of the Extreme Value Index:

Uncertainty for the Extreme Value Index

$$\hat{\xi} = \sum_{i=1}^{N} \log(1 + y_i/\phi)$$

We use this formula to estimate the tail

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Cramer-Rao Bound

$$\mathrm{var}(\theta) \geq \frac{1}{I(\theta)} \;\; ; \;\; \sqrt{N}[\hat{\theta} - \theta] \xrightarrow{d} \mathcal{N}(0, \sigma^2)$$

We obtain the estimator with least amount of variance

$$\mathrm{var}(\theta) \geq \frac{1}{I(\theta)} = \frac{1}{-(-\frac{1}{\hat{\xi}^2})} = \hat{\xi}^2 \longrightarrow \boxed{\sqrt{N}[\hat{\xi} - \xi] \xrightarrow{d} \mathcal{N}(0, \xi^2)}$$

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Example Uncertainty Estimation

$\xi_1 = -0.5$

$N = 100, N_u = 10$

95% confidence interval

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95% confidence interval

$$\hat{\xi}_1 \in [-0.65, -0.34]$$

$$\hat{q}_1 \in [176.7, 240.6]$$

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Markov’s Inequality In Low Sample Scenarios

Markov's Inequality for PTA

Markov's Inequality Theoretically

$$P(X \geq b) \leq \frac{E(X)}{b} \longrightarrow P(X \geq b) \leq \frac{E(X^k)}{b^k}$$

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Markov's Inequality for Samples

$$P(X \geq b) \leq \frac{1}{N} \frac{\sum_i^N x_i^k}{b^k} \rightarrow$$

Find the max_k for each p that does not underestimate

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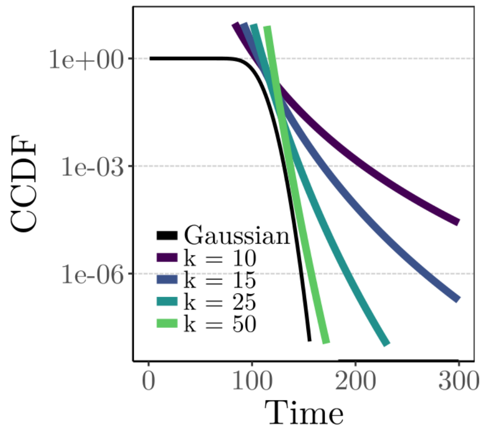
MI for pWCET

- The main consideration is that, the **higher the target quantile, the higher the *k* needed** for tight estimations
- **Higher *k* need bigger sample sizes** to estimate the moments accurately



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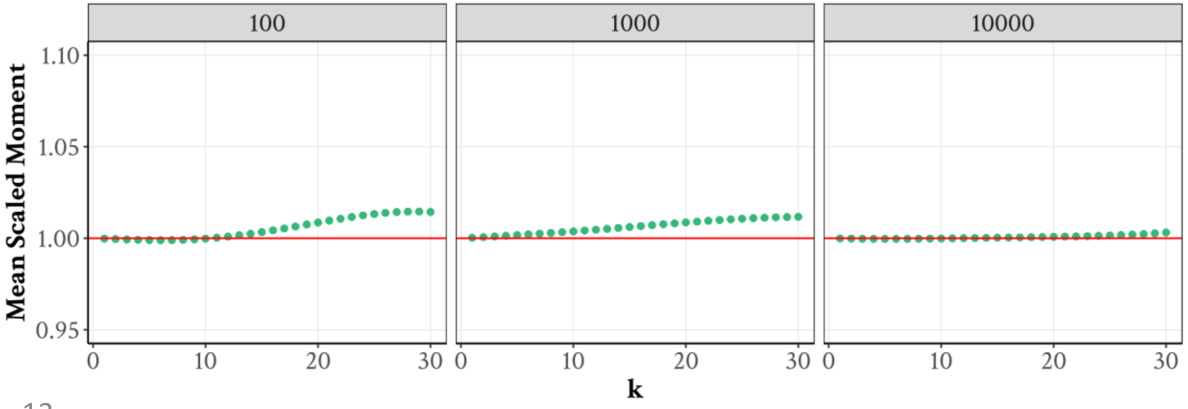
Markov's Inequality Considerations

- The estimations are done with the whole sample size, thus decreasing variance
- Markov's Inequality has been tested for sample sizes of N = 1000, but the mean scaled moment show similar results for N = 100

$$P(X \geq b) \leq \frac{\hat{E}(X^k)}{b^k} = \frac{1}{N}\frac{\sum_i^N x_i^k}{b^k}$$

$$\hat{E}(X^k)/E(X^k)$$



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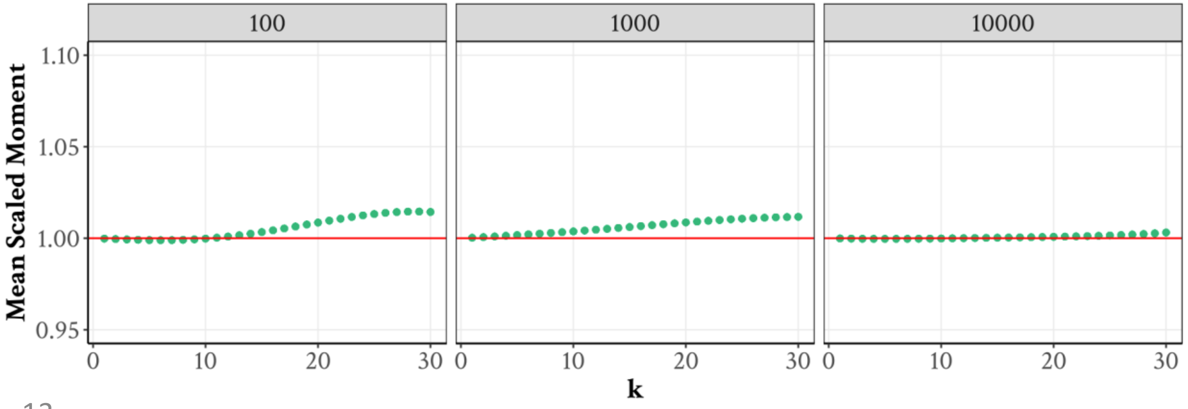
- Markov's Inequality has been tested for sample sizes of N = 1000, but the mean scaled moment show similar results for N = 100
- We aim at lower quantiles with probability $p = 10^{-6}$, thus smaller k are needed
- Need to estimate the max_k line which needs quantile estimation within the sample

$$max_k(p) = \alpha \log(p) + \beta$$

$\hat{E}(X^k)/E(X^k)$



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- We make use of the binomial Lower Confidence Interval to increase robustness in the quantile estimation

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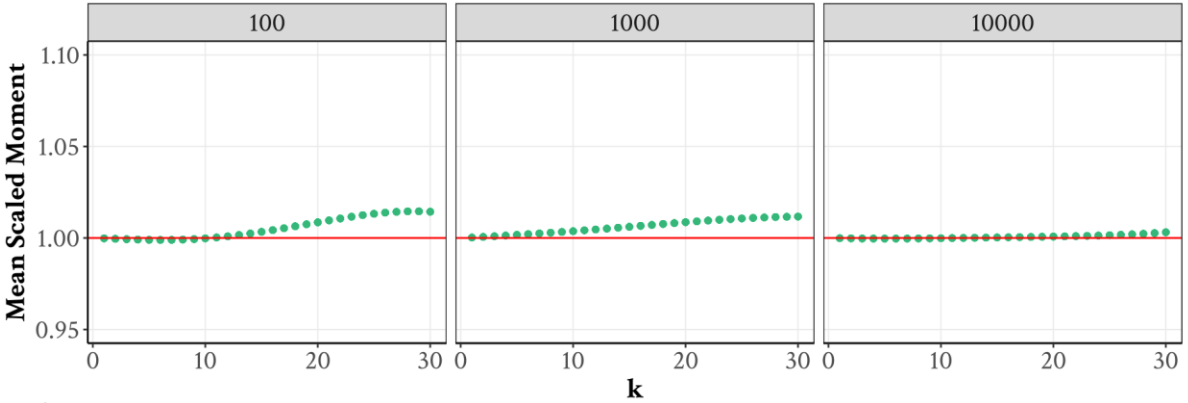

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Uncertainty Analysis Summary

- In low sample size scenarios **EVT models** are using a **very small sample size ($\mathbf{N} \sim \mathbf{10}$)** to estimate the tail
- The **least amount of uncertainty** that we can estimate (EVI estimation) is **already very high**
- **Markov Inequality** is less affected by a small sample size if the target probability is not too extreme
- The addition of the Binomial Lower Confidence Interval reduces variance
- **Let us compare EVT models and Markov's Inequality** on pWCET estimations

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Results

Results for Parametric Distributions

- **Generated synthetic mixture distributions**
- **Sample Size:** $N = 100$
- **Reference Runs:** $N_{ref} = 10^6$
- **Target Probability:** $p = 10^{-6}$

EXP/GLT

$$G(y;\sigma,\xi) = \begin{cases} 1 - \left(1 + \xi\frac{y}{\sigma}\right)^{-\frac{1}{\xi}} & \text{if} \quad \xi \neq 0, \\ 1 - \exp\left(-\frac{y}{\sigma}\right) & \text{if} \quad \xi = 0, \end{cases}$$

RESTK

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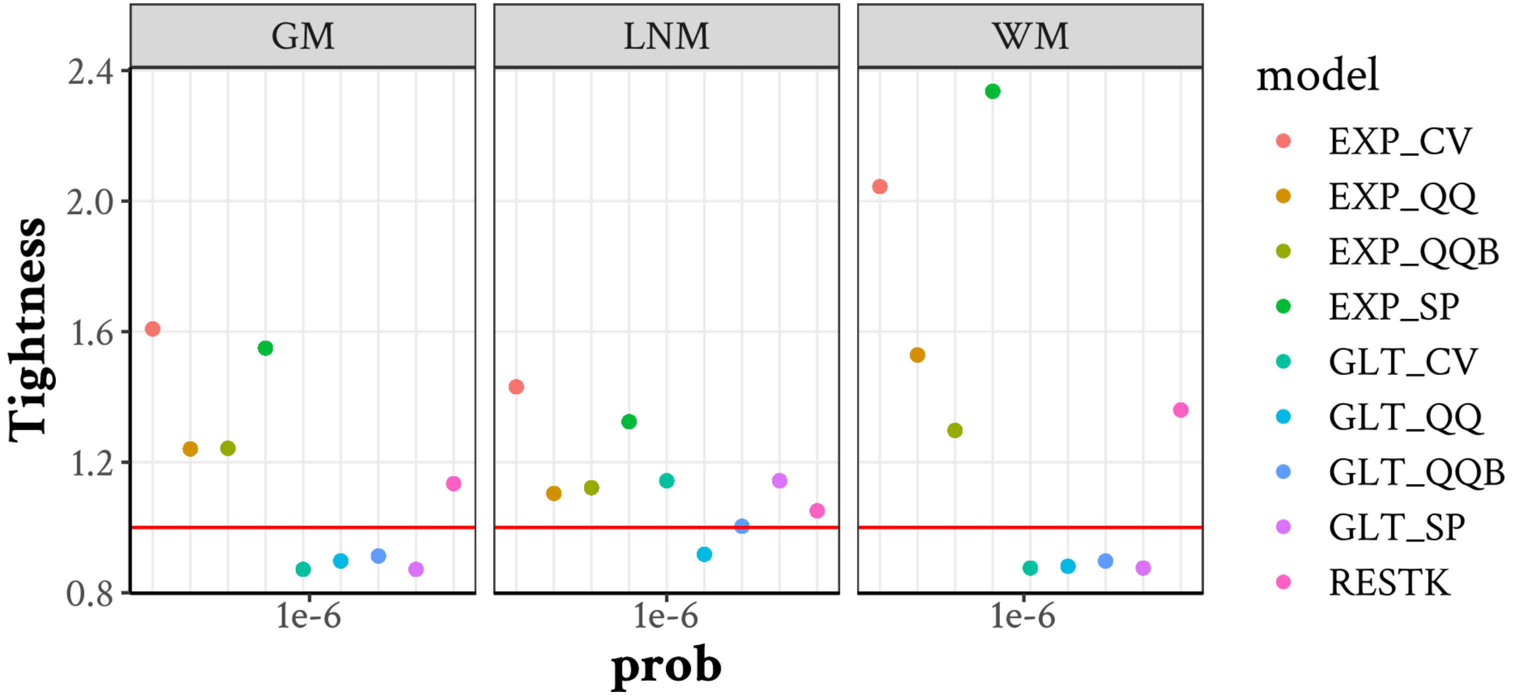

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Results for Hardware Platform

- **Platform:** NVIDIA AGX Orin
- **Sample Size:** $N = 100$
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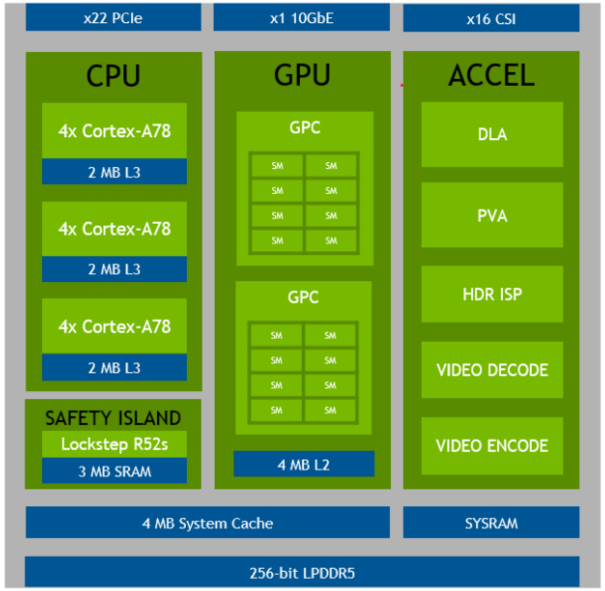

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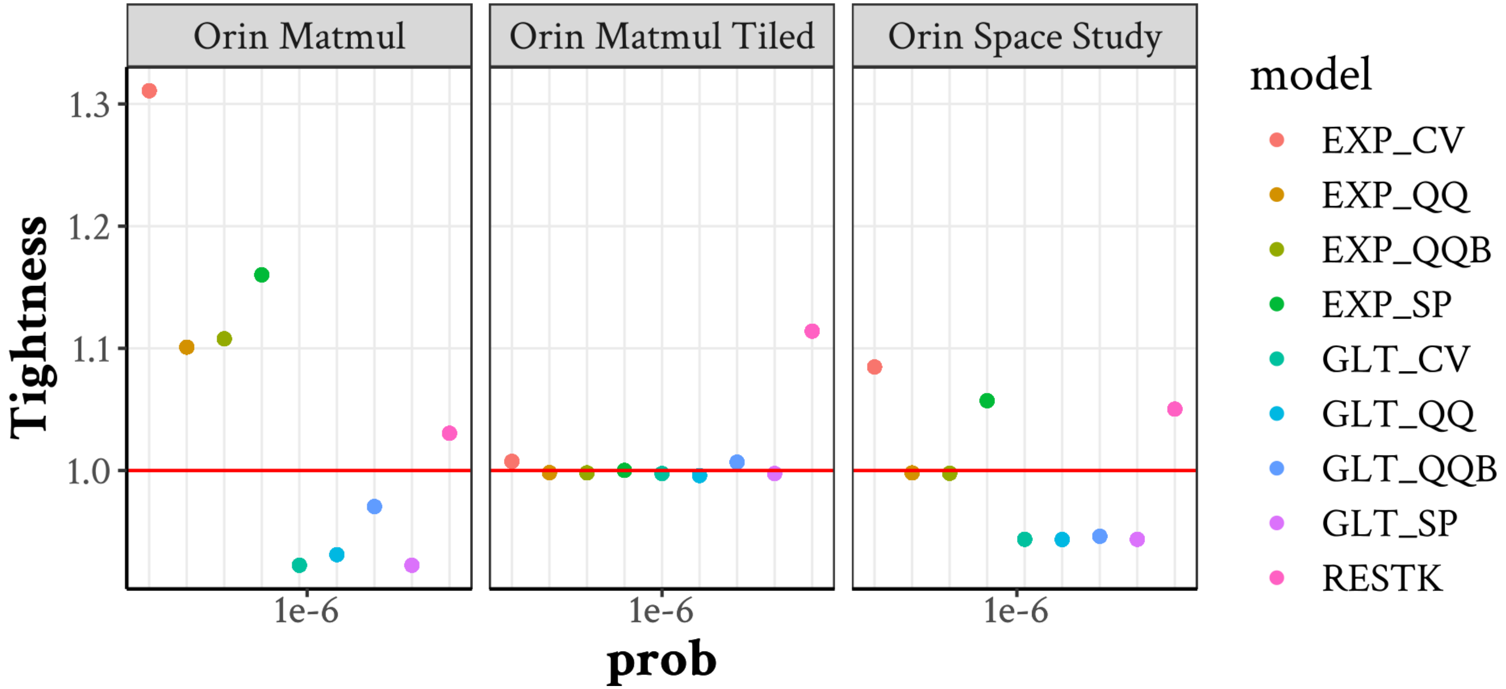

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Conclusions

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- In some domains the **risk assessment is less strict,** and costs could be reduced with few tests
- In others, risks can be **more critical** but obtaining a great number of runs is **too costly or unfeasible**
- We perform an analysis of the **effect of low sample sizes** in current MBPTA techniques

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- We perform an analysis of the **effect of low sample sizes** in current MBPTA techniques
- **EVT techniques use a fraction** of the sample for extreme modelling, thus increasing variance
- **Markov's Inequality can use the whole sample**, thus reducing variance
- We introduce the use of Binomial Lower Confidence Interval to **increase robustness** for Markov

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- We introduce the use of Binomial Lower Confidence Interval to **increase robustness** for Markov
- EVT techniques can produce **tight upperbounds**, but the results are **less consistent** due to increased variance
- The results show that Markov's Inequality can keep **consistently tight upperbounds** for a variety of scenarios in synthetic and hardware data.

Thank You

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